# Spread-Spectrum Carrier Estimation With Unknown Doppler Shift.

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## ABSTRACT

We present a method for the frequency estimation of a BPSK modulated, spread-spectrum carrier with unknown Doppler shift. The approach relies on a classic periodogram in conjunction with a spectral matched filter. Simulation results indicate accurate carrier estimation with processing gains near 40. A DSP-based prototype has been implemented for real-time carrier estimation for use in New Mexico State University's proposal for NASA's Demand Assignment Multiple Access service.

## 1. INTRODUCTION

In order for a user's spacecraft to access NASA's Tracking and Data Relay Satellite System (TDRSS), prior scheduling must be arranged with the control center. Therefore any available access time between prescheduled services is not utilized. One way to make use of the time between services is with the proposed demand assignment, multiple access (DAMA) service. The proposed DAMA service is designed to be a "911" service (and hence low- data rate) and would allow a user's spacecraft to initiate a service request which would then be scheduled in the next available time slot. At the Center for Space Telemetering and Telecommunications, we have proposed a design in which all a priori knowledge of all state vectors associated with all spacecraft which could request the DAMA service is given up for the sake of maintaining simplicity in the ground station (GS). In giving up state vector information, we can no longer calculate Doppler shift of the DAMA carrier and in general, can no longer demodulate it using current ground station receivers (GSRs). If the Doppler shift is within +/–3kHz of the nominal carrier frequency, the GS can demodulate. However, simulations have shown that the Doppler shift (between a user's spacecraft and a TDRS) could be as much as +/–50kHz. We thus must form an estimate of the carrier frequency to within +/-3kHz and pass this to the GS receiver. In this paper, we describe a method for the frequency estimation of a BPSK modulated, spread-spectrum carrier with unknown

Doppler shift and a prototype system (based around the Motorola DSP56303) for real-time implementation.

## 2. METHOD

### 2.1 Periodogram with Peak Detection

The approach to estimating the carrier frequency relies on classic, windowed-Fourier transform techniques. In this design, a length $N$ averaged periodogram [average of $M$ magnitude-squared, $N$-point discrete Fourier transforms (DFTs)] is calculated. The lower half of the periodogram is then searched (since we assume real-valued input data and hence Hermitian symmetry in the DFT) for the peak. The index, $k$ corresponding to the peak together with the sampling frequency, $f_s$ yields the estimate for the carrier frequency

$$\hat{f} = \frac{k}{N} f_s. \qquad (1)$$

Previous simulation work in the design of NMSU's proposed DAMA service, indicated that accurate (>90%) estimation of the carrier frequency (including Doppler shift) could be achieved provided the processing gain (PG) was low enough [1]. Here the PG is defined as

$$PG = \frac{R_c}{R_b} \qquad (2)$$

where $R_b$ is the data rate and $R_c$ is the chip rate. In [1] the following design parameters were used: $R_b$ = 1kbps; carrier-to-noise power ratio, $C_b / N_0$ = 45dB; and DAMA carrier frequency, $f_c$ = 2290.4MHz which is 100kHz inside the first null of TDRSS spectrum. Location of this carrier minimizes bandedge effects from the overall TDRSS system response. A maximum chip rate, $R_c$ of 100kHz was assumed (i.e. PG less than 100) and thus we have a 200kHz main lobe with the potential of being shifted a maximum of +/–50kHz. This yields a 300kHz bandwidth over which the carrier must be found. A convenient sample rate of $f_s$ = 800kHz (oversampled) was chosen assuming the signal has been shifted to baseband.

The number of periodograms to average was selected as $M = 8$. Finally, the length of the discrete Fourier transform (DFT) was chosen to be $N = 512$ so as to achieve the desired frequency resolution of less than 3kHz required for the GS receiver

$$\Delta f = \frac{f_s}{N} \tag{3}$$
$$= 1562.5\text{Hz}$$

Simulations were performed using a model of the TDRSS channel which includes other Multiple Access (MA) users and background noise and the proposed DAMA carrier with the above parameters. Figure 1 illustrates simulation data for estimation accuracy as a function of SNR. Note that $C_b / N_0 = 45$dB is approximately equivalent [based on actual White Sands Complex (WSC) data] to an SNR = 1.9dB given a typical number of other multiple-access (MA) users. In this case SNR is taken to be "DAMA carrier-to-MA users and noise ratio." The simulation data was synthesized from 10,000 frequency estimates for several values of SNR. We note a 90% (approximately) estimation accuracy rate with a $PG = 10$ and a 17% (approximately) estimation accuracy rate with a $PG = 100$.
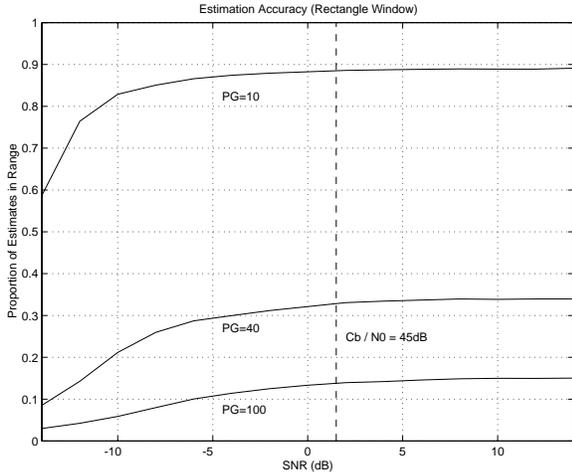


Figure 1: Estimation accuracy as a function of SNR.

## 2.2 Spectral Matched Filter

The algorithm can be enhanced to provide better estimation accuracies at higher chip rates. The enhancement is to spectrally match filter the averaged periodogram prior to the peak search. The matched filter coefficients are based on the power spectral density (PSD) of the spread-spectrum, binary phase shift keying (BPSK) signal used in DAMA. With this filtering, which serves to enhance the averaged

periodogram, we are able to accurately estimate carriers which have higher PGs (which is desirable).

The time-domain matched filter (MF) is a linear filter designed to provide the maximum signal-to-noise power ratio at its output for a given transmitted symbol waveform [2]. It can be shown that the time-domain MF is given by

$$h(t) = \begin{cases} s(T - t), & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases} \tag{4}$$

where $s(t)$ is the known signal and $T$ is the symbol time duration. Here the time-domain MF impulse response is the time-reversed (and shifted for causality) signal. Threshold comparison is then made on $r(t) * h(t)$ where $r(t)$ is $s(t)$ plus an additive noise and * indicates the convolution operation.

The MF idea may also be used to provide a maximum spectrum-to-noise power ratio. In this case we take the frequency-domain analog to (4) by first computing the power spectral density (PSD), $P_x\left(e^{j\omega}\right)$ of our known process $x(n)$

$$P_x\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} r_x(n) e^{-j\omega n} \tag{5}$$

where the correlation sequence is given by

$$r_x(n) = E\left[x(k)x(k - n)\right] \tag{6}$$

and $E[\ ]$ indicates the expectation operator [3]. The spectral matched filter (SMF) is then $P_x\left(e^{j\omega}\right)$ frequency reversed

$$H\left(e^{j\omega}\right) = P_x\left(e^{-j\omega}\right). \tag{7}$$

Finally, the peak search is performed over $\hat{P}_y\left(e^{j\omega}\right) * H\left(e^{j\omega}\right)$ where the averaged periodogram is computed from $y(n)$ equal to $x(n)$ plus an additive noise and given by [3]

$$\hat{P}_y\left(e^{j\omega}\right) = \frac{1}{NM} \sum_{i=0}^{M-1} \left|Y_i\left(e^{j\omega}\right)\right|^2. \tag{8}$$

In (8) the discrete-time Fourier transform (DTFT) of the length $N$ signal is given by

$$Y_i\left(e^{j\omega}\right) = \sum_{n=0}^{N-1} y(n + iN) e^{-j\omega n} \tag{9}$$

where $i$ indicates the block number.

For computational feasibility, we discretize the above DTFTs by sampling ω at $2\pi k / N$ where $0 \le k \le N - 1$ to form the respective DFTs and furthermore employ the fast Fourier transform (FFT). These DFTs will now be indexed by the discrete frequency variable $k$ as in

$$X\left(e^{j\omega}\right)\Big|_{\omega=2\pi k / N} = X(k). \tag{10}$$

In determining the SMF, we approximate $P_x(k)$ by simply averaging 1000 magnitude-squared FFTs of the windowed, spread-spectrum BPSK waveform, $x(n)$ using the appropriate parameters for DAMA. Although the closed form solution for $P_x(k)$ exists for rectangular windowed data, closed form solutions for other windows are more difficult. Figure 2 illustrates the SMF.
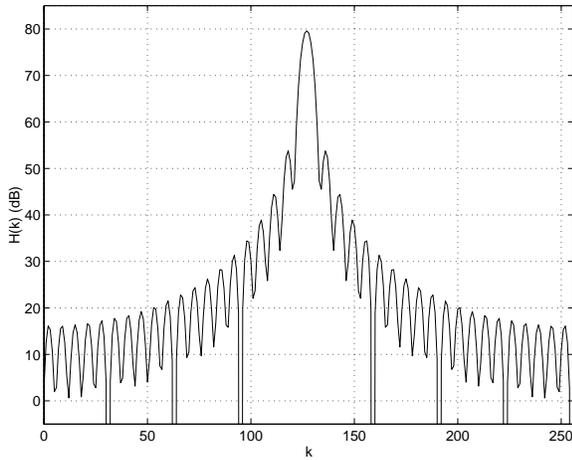


Figure 2: Spectral matched filter coefficients (matched to PSD of spread-spectrum BPSK waveform with rectangle window at $R_c = 10$kchips/s).

Since $H(k)$ and $\hat{P}_y(k)$ are both symmetric about $N / 2$, the resulting length $2N - 1$ convolution sequence (SMF output) will be symmetric about $N - 1$. Thus we need only compute the first $N$ points of the convolution. Searching the first half of the SMF output (first $N$ points) yields an index $k$ corresponding to the peak value. However, we must modify (1) to account for the offset imposed by the SMF. In this case we have as our frequency estimate

$$\hat{f} = \frac{k - (N/2)}{N} f_s$$
$$= \left(\frac{k}{N} - \frac{1}{2}\right) f_s. \tag{11}$$

## 3. SIMULATION RESULTS

Using the same design parameters in [1] and listed in Section 2.1, we now convolve the SMF given in (7) and illustrated in Figure 2 with the periodogram prior to peak searching. The carrier frequency estimate is then given by (11). In our simulation, we perform 10,000 frequency estimates to determine an estimation accuracy. We do this for various SNRs and PG keeping $R_b$ fixed to 1kbps. The results are plotted in Figures 3 and 4. Simulation results indicate that we can accurately (>90%) estimate the DAMA carrier frequency with PGs near 40. At a PG of 100 we still have about a 70% probability of accurate estimation at an SNR of 1.9dB.
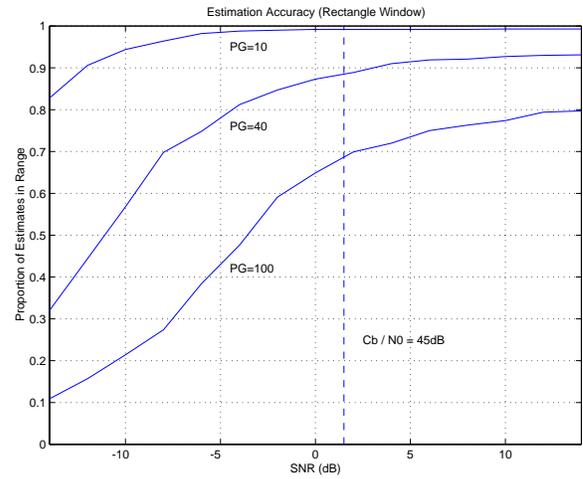


Figure 3: Estimation accuracy as a function of SNR and processing gain using a rectangle window and a spectral matched filter. $R_b = 1$kbps and $R_c = (PG)(R_b)$.
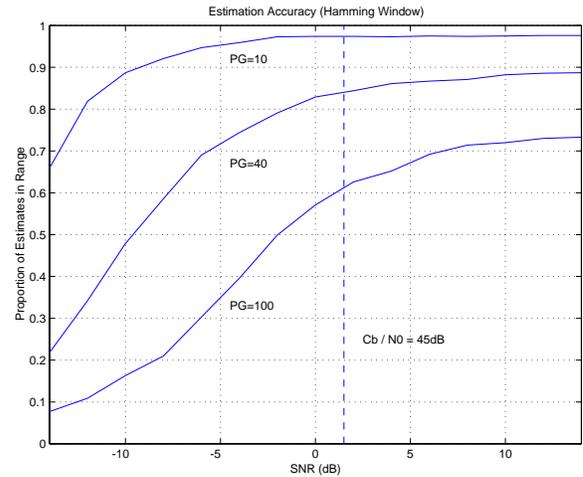


Figure 4: Estimation accuracy as a function of SNR and processing gain using a Hamming window and a

spectral matched filter. $R_b$ = 1kbps and $R_c$ = $(PG)(R_b)$.

Comparing Figure 1 to Figure 3 we note that applying the SMF prior to searching for the peak, we can now accurately (>90%) estimate the carrier with PGs near 40. At the originally proposed PG of 100 we still have about a 70% probability of accurate estimation at an SNR of 1.9dB ($C_b / N_0$ = 45dB) [4].

## 4. WHITE SANDS COMPLEX DATA

Data was collected at the White Sands Complex (NASA ground terminal) in New Mexico and used to validate the simulation model and data as well as the algorithm for frequency estimation and hardware [5]. The test procedure involved transmitting data through TDRSS with $R_b$ = 1kbps; $R_c$ = 10kbps; $C_b / N_0$ = 40, 45, 50dB; and IF frequencies (32.6, 32.65, 32.7, 32.75, 32.8MHz) to simulate carrier placement and Doppler shift. The 32.7MHz IF frequency corresponds to 2290.4MHz after down conversion. The IF (40MHz bandwidth) signal was sampled (100MHz) on a digitizing oscilloscope and stored to disk.

The estimation algorithm was run on the captured data and estimation accuracies are plotted in Figure 5. The discrete data points reflect the WSC data (at various IF frequencies) while the solid line reflects the simulation data.
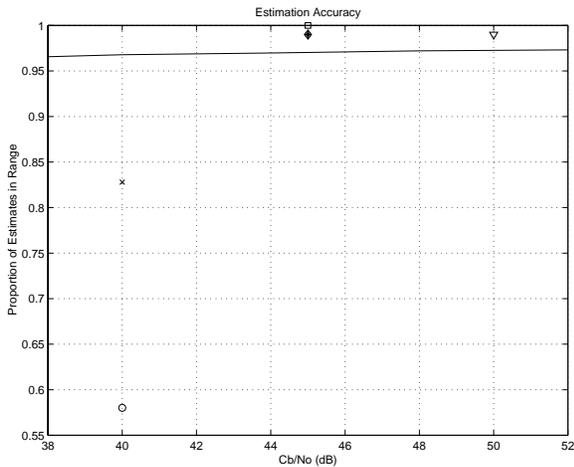


Figure 5: Estimation accuracy using WSC data.

The following results were determined:

- estimation accuracy rates using WSC data (not including $C_b / N_0$ = 40dB test set) proved to be nearly identical to simulation results thus validating the simulation model

- frequency estimation algorithm is not reliable at $C_b / N_0$ = 40dB (SNR is too low)
- frequency estimation algorithm is not reliable above 32.75MHz since bandedge effects of the TDRSS channel begin to suppress the DAMA carrier

With the above results, the WSC data supports our model and implementation of the proposed DAMA service.

## 5. PROTOTYPE DESCRIPTION

The method described above is implemented in digital hardware using an 80MHz Motorola DSP56303EVM evaluation board interfaced to a Burr Brown ADS7810/19C 12-bit, 800kHz A/D converter also on an evaluation board (ADS7810). The DSP56303EVM contains 32K of RAM partitioned into two equal banks as well as an audio-band D/A (32kHz) which is used to synthesize a locking signal to the GSR--albeit at a frequency of 1/25 the true estimate. The locking signal will be frequency multiplied by a factor of 25 to bring it up to the true baseband estimate.

The carrier estimation code is written in 100% assembly language and requires less than 1K of program memory and 6K data memory. It begins by initializing the DSP56303EVM to properly receive samples from the ADS7810 and initializes on-board memory and D/A. The algorithm then begins by filling the sample buffer with values. Once it has achieved a full block of 512 samples, it applies a windowing function of the user's choice (either rectangular or Hamming). Included in the window coefficients is a scaling factor of $1/M$ for the FFT averaging (periodogram). The code uses a Motorola-supplied FFT routine.

We compute the magnitude-squared of the FFT and repeat this step accumulating magnitude-squared to form the periodogram. Once the periodogram is computed, we convolve the result with the SMF, search for the index corresponding to the peak of the spectrum, and apply (11). A sine wave lookup table is then

used to provide a locking tone at 1/25 estimated frequency. The complete estimation process requires about 10ms from initial data acquisition until locking signal generation.

# 6. CONCLUSIONS

In this paper, we have described a method for frequency estimation of a BPSK, spread-spectrum carrier with unknown Doppler shift. The method employs a peak search on a classical periodogram convolved with a spectral matched filter. Results indicate a 90% accuracy rate at progressing gains of 40 and 70% accuracy rate at progressing gains of 100. We have developed a real-time, DSP-based implementation of the algorithm for use in NMSU's proposal for NASA's Demand Assignment Multiple Access service.

# 7. REFERENCES

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