ADVANCES IN EMPIRICAL MODE DECOMPOSITION FOR COMPUTING INSTANTANEOUS AMPLITUDES AND INSTANTANEOUS FREQUENCIES

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ABSTRACT

In this paper, we propose improvements to the Complete Ensemble Empirical Mode Decomposition (CEEMD) aimed at the resolution of closely-spaced Intrinsic Mode Functions (IMFs), reproducible and consistent decompositions, reduction in estimation error, numerical stability, and faster decompositions through fewer ensemble trials. We focus on three areas to achieve these goals: 1) use of complimentary masking signals applied at the IMF level, 2) use of narrowband tones instead of white noise for masking signals, and 3) ensuring a true IMF is obtained after ensemble averaging. We propose a numerically stable Instantaneous Frequency (IF) demodulation approach that together with a previously-reported Instantaneous Amplitude (IA) demodulation, allows estimation of the IA/IF parameters of the IMFs and hence a time-frequency representation. Using biomedical signal examples, we compare our results with CEEMD and Improved CEEMD (ICEEMD).

***Index Terms*—** Signal analysis, Empirical mode decomposition, Biomedical signal processing

1. INTRODUCTION

In a prior publication [1], we proposed a signal model consisting of a superposition of K complex, AM–FM components

$$z(t) \equiv \sum_{k=0}^{K-1} \psi_k \left(t; a_k(t), \omega_k(t), \phi_k\right) \tag{1}$$

where the component, in polar and rectangular forms, is given by

$$\psi_k(t; a_k(t), \omega_k(t), \phi_k) \equiv a_k(t)e^{j\theta_k(t)} = s_k(t) + j\sigma_k(t) \tag{2}$$

and parameterized by the Instantaneous Amplitude (IA) $a_k(t)$, Instantaneous Frequency (IF) defined through the phase derivative

$$\omega_k(t) = \frac{d}{dt}\theta_k(t), \tag{3}$$

and the phase reference ϕ_k.

Practical estimation of the instantaneous parameters of the AM–FM model in (1) is a two-step process. First, the signal must be decomposed into a set of complex AM–FM components and second, the instantaneous parameters $\{a_k(t), \omega_k(t)\}$ of each component must be estimated, i.e. demodulation. Some of the most popular AM–FM decomposition methods are Huang's Empirical Mode Decomposition (EMD) and its variations [2]. When the Hilbert Transform (HT) is used for the demodulation step, the resulting algorithm is termed the Hilbert-Huang Transform (HHT) [2].

In [2], Huang proposed the original EMD algorithm which sequentially determines a set of Intrinsic Mode Functions (IMFs), $\{\varphi_k(t)\}$ via an iterative sifting algorithm. The Ensemble Empirical Mode Decomposition (EEMD) [3] introduced ensemble averaging in order to address the mode mixing problem via an additive noise and an averaging of IMF estimates. The Complete EEMD (CEEMD) was proposed to address some of the undesirable features of EEMD by averaging at the IMF level as each IMF is estimated rather than averaging at the conclusion of EEMD [4]. The Improved CEEMD (ICEEMD) [5] was proposed to reduce the noise present in each IMF estimate and to reduce the occurrence of spurious IMFs as was observed with CEEMD. In addition, several improvements to the sifting algorithm have also been proposed including those by Rato [6]. In this paper, we propose new improvements to CEEMD including 1) a modification to the ensemble averaging which guarantees that the average IMF is a true IMF [3] and 2) a change from the additive noise used in ensemble averaging to a complimentary pair of narrowband tones which we term "tone masking."

In the context of the complex AM–FM model in (1), an IMF can be considered as the real part of the complex AM-FM component in (2), i.e. $\varphi_k(t) = \Re\{\psi_k(t)\} = s_k(t)$ when the signal under analysis, $x(t) = \Re\{z(t)\}$ [1]. In order to utilize the model in (1), the second step of the process must parameterize the IMF similarly, i.e. the IA/IF parameters must be estimated through demodulation of the IMFs. Other demodulation alternatives to the HT, used in the HHT, have been proposed to improve local behavior [6]. Rato proposed an IA estimation which is consistent with the IMF definition [6]. In addition, Huang has examined numerous alternative demodulation methods, including an iterative version of Rato's IA estimation followed by direct IF estimation. Unfortunately, this direct $\arctan$ approach suffers from numerical instability [7]. In this paper, we propose numerical stabilization techniques for Huang's iterative IA estimation and direct IF estimation algorithms. Finally, we incorporate the EMD improvements and proposed demodulation into a single algorithm which gives good estimates for the IA/IF parameters of the AM–FM model and hence a time-frequency representation.

This paper is organized as follows. In Section 2, we briefly review the EMD algorithm, the CEEMD [4], and the ICEEMD [5]. In Section 3, we describe our proposed improvements to the family of EMD algorithms as well as the demodulation procedure for IA/IF estimation. In Section 4, we compare the decompositions of example biomedical signals using the various approaches. Finally, we conclude the paper in Section 5.

2. EMPIRICAL MODE DECOMPOSITION

The purpose of the sifting algorithm is to iteratively identify and remove the trend from the signal, acting as a high pass filter. This process repeats to remove additional IMFs from the signal if they exist. The resulting decomposition is complete and sparse [2, 8, 9]. As is well-known, the major problem with EMD is *mode mixing* [3, 4, 10]. One method to mitigate mode mixing is ensemble-averaging of $\varphi_k(t)$, which leads to EEMD [3]. EEMD utilizes zero-mean white noise, $w(t)$ to perturb the signal so an IMF may be tracked properly over an ensemble average. EEMD is not without its disadvantages as it is more computationally complex, loses the perfect reconstruction property, propagates IMF estimation error, can result in inconsistent numbers of IMFs across the trials, and the resulting set of averaged IMFs $\{\bar{\varphi}_k(t)\}$ are not necessarily IMFs [4].

Torres proposed CEEMD to address some of the issues with EEMD [4]. CEEMD shifts the additive perturbation from the signal to the residue, $r(t)$ which is then sifted (see Algorithm 1) at each iteration. The resulting IMFs are then ensemble-averaged, prior to estimation of the next IMF. CEEMD defines a procedure, $\mathrm{EMD}_k(\cdot)$, which returns the kth IMF using EMD [4]; when the input is $w(t)$, it returns the kth masking signal. The result is fewer sifting iterations, a smaller ensemble size, and recovery of the completeness property of EMD to within the numerical precision of the computer [4].

Algorithm 1 Sifting Algorithm

procedure $\varphi(t) = \mathrm{SIFT}(\ r(t)\ )$
  while $\int |r(t)|^2 dt / \int |e(t)|^2 dt < \varepsilon_1$ **do**
    find all local maxima: $u_p = r(t_p),\ p = 1, 2, \ldots$
    find all local minima: $l_q = r(t_q),\ q = 1, 2, \ldots$
    interpolate: $u(t) = \mathrm{CublicSpline}(\{t_p, u_p\})$
    interpolate: $l(t) = \mathrm{CublicSpline}(\{t_q, l_q\})$
    $e(t) = [u(t) + l(t)]/2.$
    $r(t) \leftarrow r(t) - \alpha e(t).$
  end while
  $\varphi(t) = r(t)$
end procedure

The ICEEMD further attempts to improve the performance of CEEMD through a different approach to IMF estimation [5]. In the versions of EMD described thus far, the sifting algorithm directly estimates higher-frequency IMFs, i.e. "detail" from the residue. In ICEEMD, the IMF is indirectly estimated as the difference between the current residue and the average of its local means. By doing so, the sifting algorithm is replaced by an operator, $\mathrm{M}(\cdot)$, which produces the local mean of the signal that it is applied to. This potentially reduces the amount of estimation error present in the IMFs and in some cases eliminates spurious IMFs [5].

3. PROPOSED IMPROVEMENTS TO EMD AND IA/IF ESTIMATION

3.1. Improvements to EMD

Beginning with CEEMD, we propose several improvements with the following goals in mind: resolve closely-spaced IMFs, reproducible decompositions, reduction in the estimation error, numerically stable decompositions, and faster decompositions through fewer ensemble trials. Our approach prefers the direct estimate of IMFs using sifting as opposed to the indirect approach of ICEEMD. We focus in on three areas: use of complimentary masking signals at an IMF level, use of narrowband tones instead of white noise for masking signals, and ensuring a true IMF is obtained after ensemble averaging.

Complimentary EMD extends EEMD by including complimentary pairs of white noise signals, $\pm w(t)$ as the perturbation [11]. This can reduce the number of ensemble trials, I, required to guarantee that the perturbations average out of the ensemble. We propose a different approach whereby a complementary perturbation signal is applied to the residue, $r(t)$ prior to sifting. Then ensemble averaging takes place at the IMF level, as in CEEMD. We have observed that incorporation of complementary pairs of masking signals into CEEMD greatly reduces the number of trials necessary in the ensemble, thereby resulting in faster decompositions.

In EEMD and subsequent versions, the masking signal is white noise or sifted white noise. However, a deterministic signal, $v(t)$ can also be used as the perturbation [12, 13]. We propose for the kth IMF and ith trial, a narrowband masking signal with a random phase, ν drawn from $\mathcal{U}[0, 2\pi)$

$$v^{(i,k)}(t) = \beta \sin\left[c^k f_N t + \nu\right] \tag{4}$$

where $\beta > \max(|x(t)|)$ is the tone amplitude, f_N is the Nyquist frequency, and $|c| < 1$ controls the tone frequencies. In our simulations, we have observed that this approach to resolving the fundamental problem of mode mixing, has three advantages. First, a deterministic perturbation allows for reproducible decompositions including a consistent number of IMFs. Second, the number of trials can be dramatically reduced because of the complimentary pair and the resulting IMFs from sifting are not artificially noisy. Finally, this eliminates the need to use sifted white noise in CEEMD, thereby significantly reducing computation.[1]

Ensemble averaging of IMFs as in (C)EEMD, may in fact lead to $\bar{\varphi}_k$ not being an IMF itself [3, 4]. This situation is easily remedied by applying the sifting algorithm to the kth IMF estimate after the ensemble averaging, $\bar{\varphi}_k$. This step adds only a small computational burden and guarantees that the IMF estimate is a "true " IMF.

3.2. The Proposed EMD

The proposed improvements to CEEMD and demodulation are given in Algorithm 2; the demodulation in Step 6 is described next. We choose to name our algorithm Hilbert Spectral Analysis (HSA) because it represents the signal in the form of a superposition of components parameterized by IAs and IFs, although, the HT is not used.

Algorithm 2 Hilbert Spectral Analysis (HSA)

1: **procedure** $\{\hat{\varphi}_k(t), \hat{a}_k(t), \hat{\omega}_k(t)\} = \mathrm{HSA}(\ x(t)\ )$
2: initialize: $x_{-1}(t) = x(t)$, $k = 0$, β_k is a SNR factor, ε_2 is an energy threshold, and I is the number of trials
3: **while** $\int |x(t)|^2 dt / \int |x_{k-1}(t)|^2 dt > \varepsilon_2$ and $x_{k-1}(t)$ is not monotonic **do**
4: $$\bar{\varphi}_k(t) = \frac{1}{2I}\sum_{i=1}^{I}\Big[\mathrm{SIFT}(\ x_{k-1}(t) + v^{(i,k)}(t)\ ) + \mathrm{SIFT}(\ x_{k-1}(t) - v^{(i,k)}(t)\ )\Big]$$
5: $\hat{\varphi}_k(t) = \mathrm{SIFT}(\ \bar{\varphi}_k(t)\ )$
6: $[\hat{a}_k(t), \hat{\omega}_k(t)] = \mathrm{IMFdemod}(\ \hat{\varphi}_k(t)\ )$
7: $x_k(t) = x_{k-1}(t) - \hat{\varphi}_k(t)$
8: $k \leftarrow k + 1$
9: **end while**
10: $\hat{\varphi}_k(t) = x_{k-1}(t)$
11: **end procedure**

3.3. IA/IF Estimation

Rato proposed an AM demodulation approach given in Algorithm 3 [6]. Starting with an IMF estimate $\hat{\varphi}(t)$, we obtain an estimate

[1] Ensemble averaging in this approach is not to drive the perturbation to zero as in (I)(C)EEMD but rather to average out numerical effects.

for IA $\hat{a}(t)$ which can then be used to estimate the IF via the FM signal $\hat{s}_{\mathrm{FM}}(t) = \hat{\varphi}(t)/\hat{a}(t)$. However, this estimate may result in $|\hat{s}_{\mathrm{FM}}(t)| > 1$. Thus, Huang proposed an iterative normalization procedure, given in Algorithm 4, which removes the AM from the signal to obtain a more accurate $\hat{s}_{\mathrm{FM}}(t)$ [7].

Direct FM demodulation is not straightforward because different approaches exist for obtaining the IF from $\hat{s}_{\mathrm{FM}}(t)$, that although are mathematically equivalent, may differ in numerical stability [7]. Huang proposed to estimate the phase, $\hat{\theta}(t)$, from the normalized FM signal, $\hat{s}_{\mathrm{FM}}(t)$, as $\hat{\theta}(t) = \arg\left(\hat{s}_{\mathrm{FM}}(t)/[1 - \hat{s}^2_{\mathrm{FM}}(t)]\right)$ and then IF is obtained with (3).

Algorithm 3 IA Estimation

1: **procedure** $\hat{a}(t) = \mathrm{IAest}(\ \hat{\varphi}(t)\ )$
2: $\quad r(t) = |\hat{\varphi}(t)|$
3: $\quad$ find all local maxima: $u_p = r(t_p),\ p = 1, 2, \ldots$
4: $\quad$ interpolate: $\hat{a}(t) = \mathrm{CublicSpline}(\{t_p, u_p\})$
5: **end procedure**

Algorithm 4 Obtaining a real FM signal from an IMF

procedure $\hat{s}_{\mathrm{FM}}(t) = \mathrm{iterAMremoval}(\ \hat{\varphi}(t)\ )$
$\quad$ initialize: $g(t) = \hat{\varphi}(t)$, $b(t) \neq 1$, and $n = 0$
$\quad$ **while** $b(t) \neq 1$ and $n < 3$ **do**
$\quad\quad b(t) = \mathrm{IAest}(\ g(t)\ )$
$\quad\quad g(t) \leftarrow g(t)/b(t)$
$\quad\quad n \leftarrow n + 1$
$\quad$ **end while**
$\quad \hat{s}_{\mathrm{FM}}(t) = g(t)$
end procedure

3.4. Improvements to IA/IF Estimation

Although the iterative procedure improves AM demodulation accuracy, it can be susceptible to oscillating artifacts introduced by overfitting of the cubic spline interpolator. As a result, we have found that these artifacts can be minimized by replacing the cubic spline interpolation (Step 4 in Algorithm 3) with a Piecewise Cubic Hermite Interpolating Polynomial (PCHIP). PCHIP has no overshoot and less oscillation, thereby increasing the stability of IA estimation.

In order to address the numerical stability issues associated with FM demodulation, we begin by estimating the quadrature in (2) as

$$\hat{\sigma}_{\mathrm{FM}}(t) = -\mathrm{sgn}\left[\frac{d}{dt}\hat{s}_{\mathrm{FM}}(t)\right]\sqrt{1^2 - \hat{s}^2_{\mathrm{FM}}(t)} \tag{5}$$

where $-\mathrm{sgn}\left[\frac{d}{dt}\hat{s}_{\mathrm{FM}}(t)\right]$ is required to obtain an appropriate four quadrant estimate with assumed positive IF. This intermediate step allows for more stable estimations because the computationally unstable points, $\{t_0\}$ occur near $\hat{\sigma}_{\mathrm{FM}}(t_0) = 0$. Thus, we can replace a small range around these points $(t_0 - \epsilon, t_0 + \epsilon)$ with interpolated values. Then $\hat{\theta}(t) = \arg[\hat{s}_{\mathrm{FM}}(t) + j\hat{\sigma}_{\mathrm{FM}}(t)]$ and the IF is obtained with (3). Our IMF demodulation is listed in Algorithm 5.

In summary, to estimate the IA/IF parameters of the signal $x(t)$ using the decomposition model in (1) with complex AM-FM components given in (2), use Algorithm 2 which computes and IMF decomposition and demodulates the components using Algorithm 5.

4. EXAMPLES

In this section, we compare the decomposition results of the CEEMD, ICEEMD, and our proposed decomposition algorithm

Algorithm 5 IMF demodulation

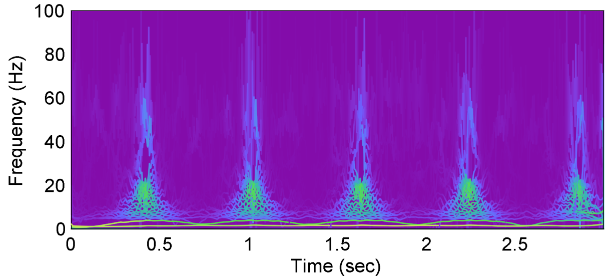
procedure $[\hat{a}(t), \hat{\omega}(t)] = \mathrm{IMFdemod}(\ \hat{\varphi}(t)\ )$
$\quad \hat{a}(t) = \mathrm{IAest}(\ \hat{\varphi}(t)\ )$
$\quad \hat{s}_{\mathrm{FM}}(t) = \mathrm{iterAMremoval}(\ \hat{\varphi}(t)\ )$
$\quad \hat{\sigma}_{\mathrm{FM}}(t) = -\mathrm{sgn}\left[\frac{d}{dt}\hat{s}_{\mathrm{FM}}(t)\right]\sqrt{1^2 - \hat{s}^2_{\mathrm{FM}}(t)}$
$\quad$ Find $\{t_0\}$ such that $\hat{\sigma}_{\mathrm{FM}}(t_0) = 0$
$\quad$ For each t_0, replace $(\hat{\sigma}_{\mathrm{FM}}(t_0 - \epsilon), \hat{\sigma}_{\mathrm{FM}}(t_0 + \epsilon))$ with interpolation
$\quad \hat{\omega}(t) = \frac{d}{dt}\arg[\hat{s}_{\mathrm{FM}}(t) + j\hat{\sigma}_{\mathrm{FM}}(t)]$
end procedure

by plotting the IA/IF parameters using our proposed demodulation algorithm. We utilize a 2D visualization of the Hilbert spectrum as proposed in [14] which plots IA vs. IF vs. time. Color variation in the plot line indicates the IA of the IMF while the value of the plot line along the frequency axis indicates the IF of the IMF. In addition, we include a plot of the Short-Time Fourier Transform (STFT) for comparison. We use two real-world, test signals. The first is the Electrocardiogram (ECG) signal from the MIT-BIH Normal Sinus Rhythm Database [15] which we upsample by $2\times$ to $f_s = 256$ Hz and the second is a Phonocardiogram (PCG) from the Classifying Heart Sounds Challenge [16] sampled at $f_s = 44.1$ kHz.

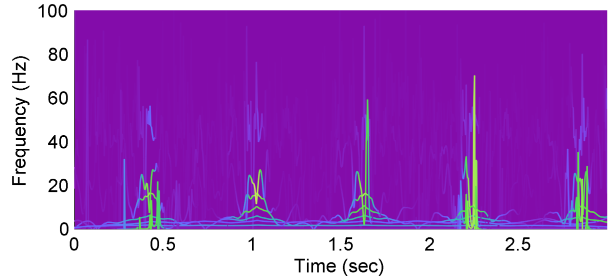
For comparisons, we use reference codes for CEEMD and ICEEMD found at [17]; our HSA code can be found at [18]. For CEEMD, ICEEMD, we set $I = 500$, $I = 50$, respectively, and the noise standard deviation parameter to 0.2 which are all similar to the authors' choices in [4, 5]. For the HSA algorithm, we choose $\alpha = 0.95$, $\beta = 3$, $\varepsilon_1 = 30$ dB, $\varepsilon_2 = 10$ dB, $c = 0.9$, and $I = 20$ (40 actual masking signal realizations). For all algorithms, we limit the maximum number of iterations in sifting to 50. In order to strike a balance between time and frequency resolution, the STFT uses a 500 ms Hamming window for the ECG signal and 30 ms for the PCG signal. The time-frequency decompositions for the ECG are shown in Fig. 1 and for the PCG are shown in Fig. 2.

For the ECG signal, the HSA algorithm, ICEEMD, and CEEMD returned 27, 10, and 9 IMFs respectively which are shown in Fig. 1. Close examination of Fig. 1(a) reveals that the HSA algorithm is capable of decomposing multiple IMFs within a single octave, which is in contrast to empirical experiments with EMD using white noise that have shown EMD to act as a dyadic filter bank [3, 19–22]. Figs. 1(b) and (c) mirror these empirical experiments. In this example, both CEEMD and ICEEMD also exhibit numerical instability in one of more IMF estimations: demodulation of these IMFs leads to strong, sharp "spikes" in the time-frequency plane [see for example $t = 0.5$ and $t = 2.3$ in Fig. 1(b) and $t = 1$, $t = 1.8$, and $t = 2.3$ in Fig. 1(c)]. The ECG signal is quasi-periodic and thus we expect some periodicity in the time-frequency decomposition. This property is more apparent in the HSA algorithm in Fig. 1(a) than for the ICEEMD and CEEMD. Finally, we note the higher level of resolution in the three EMD-based techniques than with the STFT.

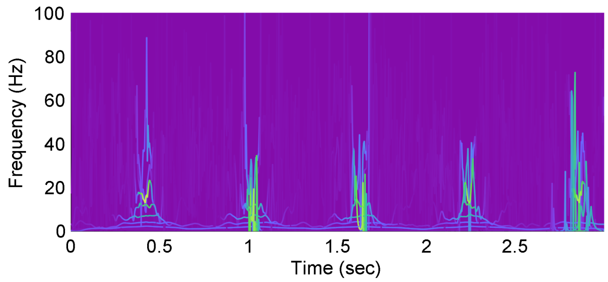
For the PCG signal, the HSA algorithm, ICEEMD, and CEEMD, returned 12, 19, and 20 IMFs, respectively which are shown in Fig. 2. This demonstrates that the HSA algorithm is capable of returning a more compact representation than (I)CEEMD. This is unlike the ECG example where we had intra-octave IMFs present that the HSA algorithm was able to resolve. This example also illustrates numerical instability in one of more IMF estimations [see for example $t = 1.8$ and $t = 2.6$ in Fig. 2(b) and $t = 0.2$ and $t = 1$ in Fig. 2(c)]. Finally, we see in Fig. 2(a) that the HSA algorithm aligns better with STFT in Fig. 2(d) but with much better time-frequency resolution.



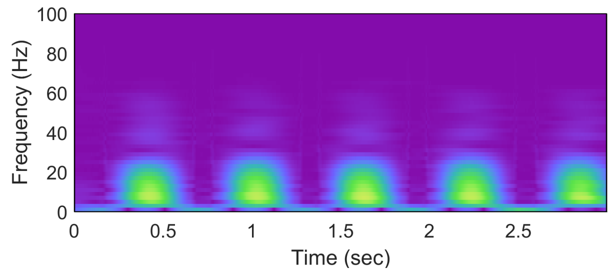
(a) Proposed algorithm



(b) Improved Complete Ensemble EMD (ICEEMD)

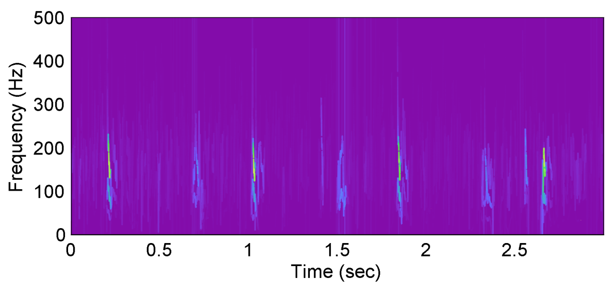


(c) Complete Ensemble EMD (CEEMD)

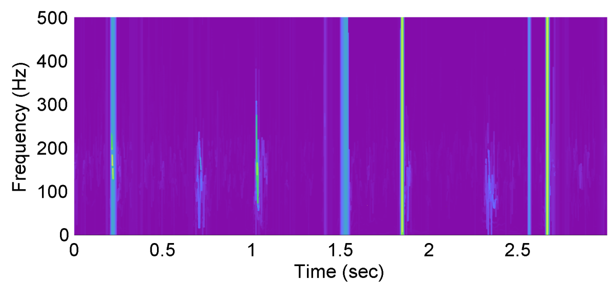


(d) Short-Time Fourier Transform (STFT)

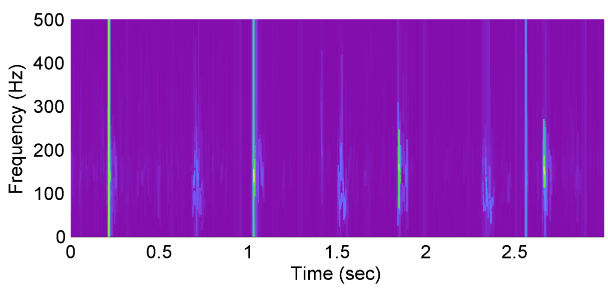
Fig. 1. Time-frequency decompositions of the electrocardiogram (ECG) signal using: (a) the HSA algorithm; (b) ICEEMD; (c) CEEMD; and (d) STFT. For (a)-(c) the IA/IF estimation (demodulation) of IMFs is computed using Algorithm 5.



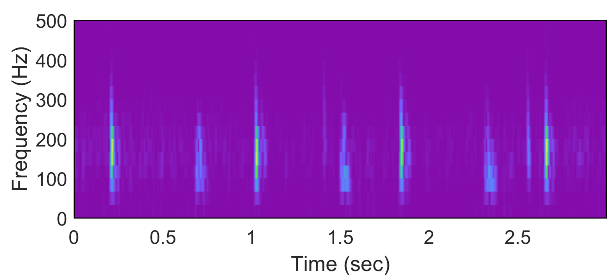
(a) Proposed algorithm



(b) Improved Complete Ensemble EMD (ICEEMD)



(c) Complete Ensemble EMD (CEEMD)



(d) Short-Time Fourier Transform (STFT)

Fig. 2. Time-frequency decompositions of the Phonocardiogram (PCG) signal using: (a) the HSA algorithm; (b) ICEEMD; (c) CEEMD; and (d) STFT. For (a)-(c) the IA/IF estimation (demodulation) of IMFs is computed using Algorithm 5.

5. CONCLUSION

In this paper, we proposed an algorithm consisting of several improvements to the CEEMD algorithm, including complimentary tone masking, an additional sift of the ensemble estimate, and a numerically stable IF demodulation. We computed and visualized the Hilbert spectrum of two biomedical signals using the HSA algorithm and compared the result to CEEMD, ICEEMD, and STFT. The HSA algorithm led to improvements in the ability to resolve closely-spaced IMFs, reproducibility and consistency of the decompositions, and computation. In addition, overall numerical stability was improved and computation was reduced.

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